Using Steady State CFD for Acoustic Predictions

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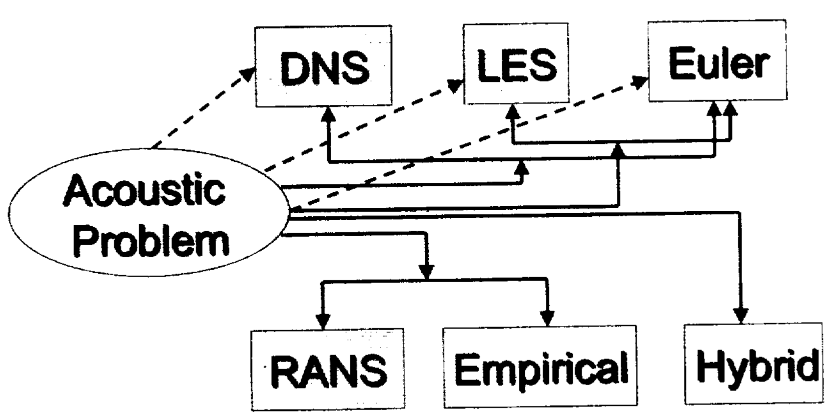
Background



Empirical Model (1)

- Far-field Approximation to The Lighthill Acoustic Analogy

$$p(R,t)=\frac{R_i R_j}{4\pi C_\infty^2 R^3}\int_V\left[\frac{\partial^2 T_{ij}}{\partial t^2}\right]d^3 r$$

where

$$T_{ij}=\rho v_i v_j+\tau_{ij}+(\pi-C_\infty^2\rho)\delta_{ij}$$

Empirical Model (2)

Steady State CFD $\Rightarrow$ No Time History

$$\overline{p^2}(R,\theta,\phi)=\frac{R_i R_j R_k R_l}{16\pi^2 C_\infty^4 R^6}\int_V\int_V\frac{\overline{\partial^2(\rho v_i v_j)\partial^2(\rho v'_k v'_l)}}{\partial t'^2\partial t''^2}d^3y' d^3y''$$

Which becomes

$$\overline{p^2}(R,\theta,\phi)=\frac{\rho^2 R_i R_j R_k R_l}{16\pi^2 C_\infty^4 R^6}\int_V\frac{\partial^4}{\partial\tau^4}\overline{v_i v_j v'_k v'_l}\,d^3 r$$

Empirical Model (3)

Using

$$S_{ijkl} = \overline{v_i v_j v_k v_l} = S_{ik}S_{jl} + S_{il}S_{jk} + S_{ij}S_{kl}$$

and assuming

$$S_{ij}(r,\tau) = \Lambda_{ij}(r)g(\tau)$$

with

$$\Lambda_{ij}(r) = k\left[\left(f + \frac{1}{2}rf'\right)\delta_{ij} - \frac{1}{2}f'\frac{r_i r_j}{r}\right]$$

Empirical Model (4)

where

$$f(r) = e^{-\pi^2 r^2 / L^2} \qquad \text{and} \qquad g(\tau) = e^{-(\tau/\tau_0)^2}$$

Define

$$\tau_{0,U} = \left(c_t^U \frac{\partial U}{\partial n}\right)^{-1} \quad \text{with} \quad \frac{\partial U}{\partial n} = \sqrt{\left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2}$$

Empirical Model (5)

In addition

$$\tau_{0,\varepsilon} = c_t^{\varepsilon} \frac{k}{\varepsilon} \qquad \text{and} \qquad \tau_{0,\mathrm{Pr}} = c_t^{\mathrm{Pr}} \frac{k}{\mathrm{Pr}}$$

Define

$$L = c_l k^{1/2} \tau_{0,x}$$

The spectrum is given by

$$I(\Omega) = c_l^3 \rho^2 k^{7/2} (\Omega \tau_{0,x})^4 e^{-\frac{1}{8}(\Omega \tau_{0,x})^2}$$

Empirical Model (6)

where

$$\Omega = 2\pi f \sqrt{(1 - M_c \cos\theta)^2 + (c_d k^{1/2} / C_\infty)^2}$$

with

$$M_c = \frac{1}{2} M + c_c M_{jet}$$

Empirical Model (7)

Finally

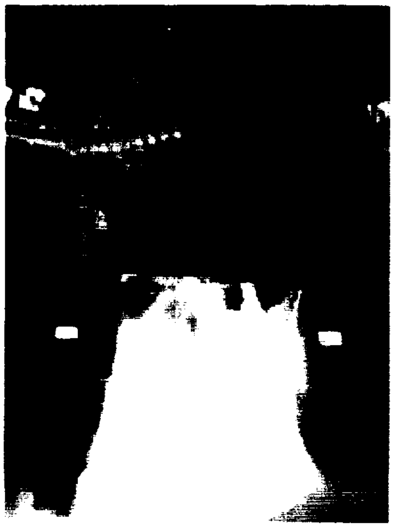
$$\overline{p^2}(R,\theta,\Omega) = \int_V (a_{xx} + 4a_{xy} + 2a_{yy} + 2a_{yz})\Psi d^3 r$$

with

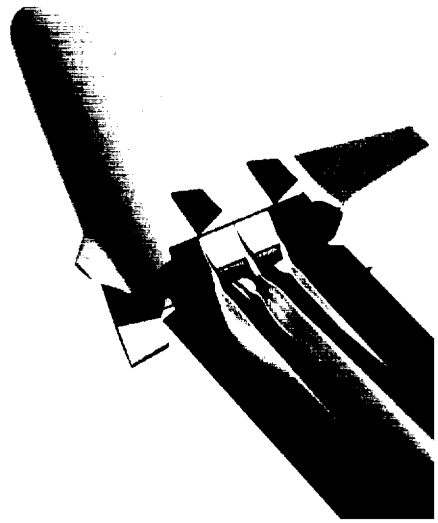
$$\Psi = \frac{I(\Omega)}{16\pi^2 R^2 C_\infty^4}\left(\frac{\rho_\infty}{\rho}\right)^2\left(\frac{C_\infty}{C}\right)^2(1 - M\cos\theta)^{-2}(1 - M_c\cos\theta)^{-1}$$

Results

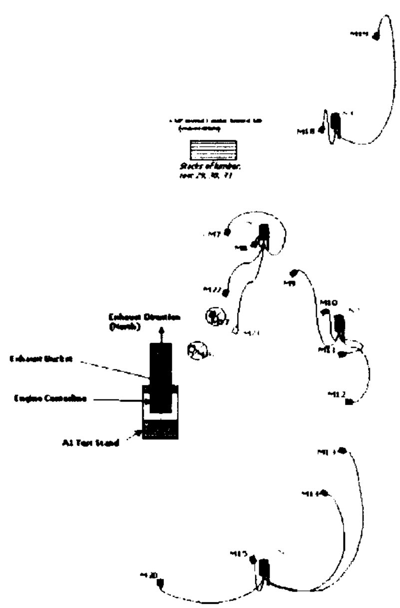
Hotfire Test



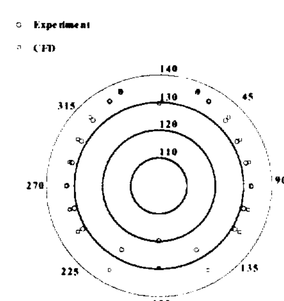
CFD Solution



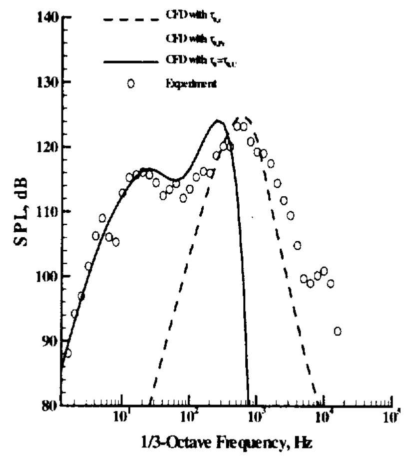
Mid-field locations

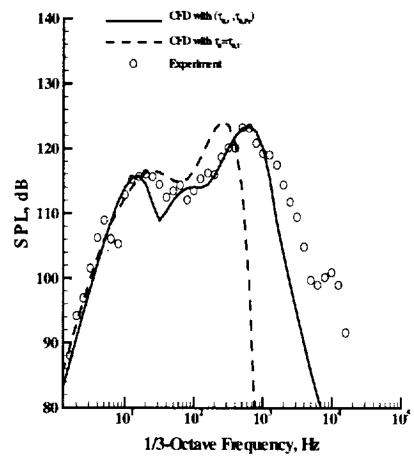


OASPL

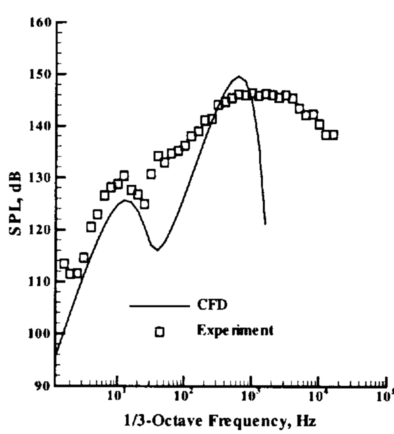


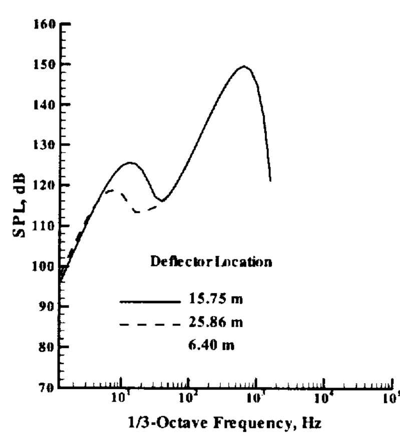
SPL at 300' from the test stand and at 90° from the deflected plume



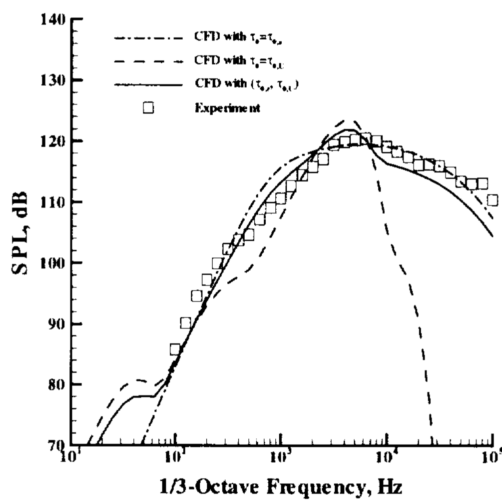


SPL on the test stand between the primary and deflected plumes





Axi-symmetric Jet Plume



Conclusions

- A semi-empirical model for acoustic predictions has been presented.
- The predicted midfield directivity pattern is in good agreement with the measurements
- The predicted midfield spectrum at 90° is In good agreement with that measured.
- The results showed the importance of using two time scales in the model.